

SPIN OBSERVABLES OF (d, p) REACTIONS OFF ^{12}C WITHIN THE PARIS POTENTIAL

T. V. Nhan Hao^{1,2*}, Vinh N. T. Pham³, H. Khac Nguyen^{1,2}

¹ Faculty of Physics, University of Education, Hue University, 34 Le Loi St., Hue, Vietnam

² Center for Theoretical and Computational Physics, University of Education, Hue University, 34 Le Loi St., Hue, Vietnam

³ Department of Physics, Ho Chi Minh City University of Education, 280 An Duong Vuong St., District 5, Ho Chi Minh City, Vietnam

* Correspondence to Tran Viet Nhan Hao <tvnhao@hueuni.edu.vn>

(Received: 16 October 2019; Accepted: 11 November 2019)

Abstract. Deuteron elastic scattering off ^{12}C is described in the framework of three-body Faddeev-type equations. Analyzing powers are calculated using the PEST16 potential, which is a separable rank-one representation of the Paris potential. Satisfactory agreement with the experimental data is found within the PEST16 potential.

Keywords: (d, p) reactions, Faddeev-AGS equations, Paris potential

Deuteron induced-reactions (d, p) are the simplest transfer reaction. This reaction is attractive from the experimental perspective since the deuterated targets are readily available. In inverse kinematics, the (d, p) reaction provides a useful tool for extracting the nuclear information of the neutron capture needed for nuclear astrophysics and applied physics calculations. If we consider the target as an inert core, the complicated $A + 2$ problem could be reduced to the three-body problem. In the literature, there are some approaches to solve this three-body problem, such as Distorted Wave Born Approximation (DWBA), Adiabatic Distorted Wave Approximation (ADWA), Continuum Discretized Coupled-Channels method (CDCC), and Faddeev/Alt, Grassberger, and Sandhas equations (FAGS). Among these approaches, FAGS is the unique approach that could exactly and simultaneously describe the elastic, inelastic, transfer, and breakup reactions. Then, the FAGS calculations could be the

benchmark for the rest of the three-body approaches [1].

The correct description of the analyzing power is one of the most challenging tasks for nucleon-nucleus scattering in general, especially for (d, p) reactions, using the exact three-body FAGS equations. Early calculations [2] use a simple separable NN interaction of Phillips [3] for NN and potential of Miyagawa and Koike [4] for NA interaction to reproduce the polarisation of deuteron elastic scattering off ^{12}C . They show, for the analyzing power, strong disagreement with experimental data at $E_d = 56$ MeV. They suggest that the main reason is that the lowest contributing spin-single P state has not been taken into account since the Phillips potential acts in the coupled 3S_1 - 3D_1 states only. The most performance code of Deltuva et al. [5] uses the realistic CD Bonn potential [6] for the NN interaction of Watson et al. [7] and Menet et al. [8] (including spin-orbit interaction) for the NA interaction. Differential

cross-section and analyzing power for the deuteron elastic scattering for light nuclei have been performed. They also fail to reproduce the vector analyzing power A_y at small angles for both ^{12}C and ^{16}O at $E_d = 56$ MeV as functions of the c.m. scattering angle. The goal of this article is to improve the calculations of Ref. [2] by taking the PEST16 potential [9], which is the separable form of the Paris potential [10].

In the literature, the FAGS equations have been extensively used to study the (d, p) reactions. First, let us briefly recall some general features of these equations. According to Ref. [2], we consider a system consisting of two nucleons (1 proton and 1 neutron) and a nucleus consisting of A nucleons. In principle, scattering of deuteron off a nuclear target can be exactly described by the FAGS equation

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma=1}^3 \bar{\delta}_{\beta\gamma} T_\gamma G_0 U_{\beta\gamma} \quad (1)$$

where $U_{\beta\alpha}$ is the transition operator from the initial channel $\alpha + (\beta\gamma)$ to the final channel $\beta + (\alpha\gamma)$; $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$ is the anti-Kronecker symbol; $G_0 = (E + i0 - H_0)^{-1}$ is the free resolvent; E is the available three-particle energy in the center of the mass system; H_0 is the free Hamiltonian; $T_\gamma = V_\gamma + V_\gamma G_\gamma V_\gamma$ is the transition operators of the two-particles systems; V_γ is the potential for the pair γ in odd-man-out notation. The channel states $|\Phi_\gamma\rangle$ are the eigenstates of the corresponding channel Hamiltonian $H_\gamma = H_0 + V_\gamma$.

If the excited states of the core must be taken into account, the generalization of the FAGS equations has been proposed by Ref. [2]. Suppose a system of three particles interacting by the pairwise interactions V_1 (V_2), which describes the interaction of particles 2 and 3 (particles 1 and 3), respectively. The potential $V_3 = V_3^S + V_3^C$, where V_3^S and V_3^C are the short-range and the Coulomb part of the proton target interaction, respectively.

The Hamiltonian of the three-body system reads

$$H = H_{\text{int}} + H_0 + V, \quad \text{where } H_0 = \mathbf{K}_\alpha^2/2\mu_\alpha + \mathbf{Q}_\alpha^2/2M_\alpha, \quad \text{and } V = V_1 + V_2 + V_3. \quad (2)$$

where H_{int} is the internal Hamiltonian of nucleus 2; H_0 is the Hamiltonian of the relative motion of the non-interacting particles 1, 3 and the center of mass of particle 2; \mathbf{K}_α is the momentum operator for the relative motion of particles β and γ and $\mu_\alpha = m_\beta m_\gamma / m_{\beta\gamma}$ the corresponding reduced mass, $m_{\beta\gamma} = m_\beta + m_\gamma$; \mathbf{Q}_α is the relative momentum operator for the motion of particle α and the center of mass of (β, γ) with $M_\alpha = m_\alpha m_{\beta\gamma} / (m_\alpha + m_{\beta\gamma})$.

Consider ρ ($\rho = 1, 2, \dots, N$) are the excited states of the target. The corresponding wave functions $|\varphi^\rho\rangle$ and energies $\varepsilon^\rho \geq 0$ satisfy

$$H_{\text{int}}|\varphi^\rho\rangle = \varepsilon^\rho|\varphi^\rho\rangle. \quad (3)$$

To reduce the many-body problem to the three-body problem, all the operators acting in the $(A+2)$ -particle space are projected onto the three-particle space

$$\underline{H} = [H^{\rho\sigma}] = [\langle \varphi^\rho | H | \varphi^\sigma \rangle], \quad (4)$$

$$\underline{H}_0 = [H_0^{\rho\sigma}] = [\langle \varphi^\rho | H_0 | \varphi^\sigma \rangle] = [\delta_{\rho\sigma} (\mathbf{K}_\alpha^2/2\mu_\alpha + \mathbf{Q}_\alpha^2/2M_\alpha)], \quad (5)$$

$$\underline{V}_\alpha = [V_\alpha^{\rho\sigma}] = [\langle \varphi^\rho | V_\alpha | \varphi^\sigma \rangle], \quad (6)$$

$$\text{and } [\langle \varphi^\rho | \mathbf{Q}_\alpha | \varphi^\sigma \rangle] = [\delta_{\rho\sigma} \mathbf{Q}_\alpha]$$

and $[\langle \varphi^\rho | \mathbf{K}_\alpha | \varphi^\sigma \rangle] = [\delta_{\rho\sigma} \mathbf{K}_\alpha]$. The resolvent matrices corresponding to the restricted full and free Hamiltonian matrices are

$$\mathcal{G}(z) = (z - \underline{H})^{-1}, \quad \text{and } \mathcal{G}_0(z) = (z - \underline{H}_0)^{-1}. \quad (7)$$

After the projection, as shown in Ref. [2], we obtain the modified FAGS equation

$$\underline{U}_{\beta\alpha}(z) = \bar{\delta}_{\beta\alpha} \underline{G}_0^{-1}(z) + \sum_\gamma \bar{\delta}_{\gamma\alpha} \underline{U}_{\beta\gamma}(z) \underline{G}_0(z) \underline{T}_\gamma(z), \quad (8)$$

where \underline{U} is the three-body modified transition operators; $\underline{G}_0(z)$ is the three-body modified free Green's function; $\underline{t}_\gamma(z)$ is the projection of the two-body T operators satisfying the Lippmann-Schwinger equation

$$\underline{t}_\alpha(z) = \underline{V}_\alpha + \underline{V}_\alpha \underline{G}_0(z) \underline{t}_\alpha(z). \quad (9)$$

The Paris potential is well known since it yields a good reproduction of both on and off-shell NN data. However, due to the complicated form of the original version, it has not yet been possible to use it for the three-body calculations. Therefore, the separable form of this potential (PEST16 version) has been proposed and successfully applied to the p-d scattering at low-energy [9, 11].

The calculations have been performed for the elastic-scattering process $d + {}^{12}\text{C} \rightarrow d + {}^{12}\text{C}$ and the transfer reaction $d + {}^{12}\text{C} \rightarrow p + {}^{13}\text{C}$ at different incident deuteron energies. Since the scope of this paper is to improve the analyzing power of the work of Ref. [2], we focus on the analyzing power of elastic scattering only. For the given value of J^Π (J is the total angular momentum, and Π is the parity of the three-body

system), the maximum number of coupled channels is 76 at 56 MeV. To take into account the Coulomb repulsion between the charged particles in the three-body system, we adopt the method proposed by Ref. [2]. For the NA interaction, we use the parameters of Ref. [4] for $J^\Pi = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}, \frac{7}{2}^{\pm}, \frac{9}{2}^{\pm}, \frac{11}{2}^{\pm}$ as in Ref. [2]. The parameters of this potential have been obtained by fitting the experimental data of elastic scattering of protons from several 1p-shell nuclei (including ${}^{12}\text{C}$) in the energy interval 10–50 MeV.

We adopt the PEST16 for the NN interaction. Fig. 1 shows the systematic calculations of the vector polarization for deuteron elastic scattering off ${}^{12}\text{C}$ at 12, 29.5, 35, and 56 MeV by using the PEST16 potential. The obtained results have been compared with the same calculations using the Phillips potential and the experimental data. The results show the important role of the quality of the NN interaction in describing the analyzing power of this reaction. However, the deficiency of the analyzing power at low-energies reflects the limitation of the using NA interaction.

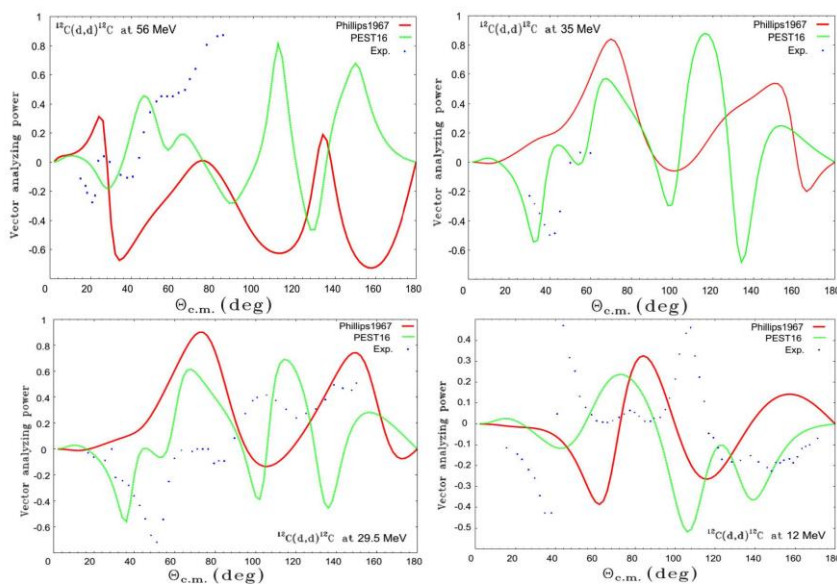


Fig. 1. Analyzing powers of (d, p) reactions using the Faddeev-AGS equations within PEST16 potential at different incident energies. The experimental data are taken from Ref. [12]

Funding statement

This research is funded by the Vietnam Ministry of Education and Training (MOET) under Grant No. B2019-DHH-14.

Acknowledgments

T. V. Nhan Hao thanks Profs. A. M. Mukhamedzhanov, A. I. Sattarov, and C. Bertulani for the useful discussions and their supports.

References

1. Nunes FM, Deltuva A. Adiabatic approximation versus exact Faddeev method for (d,p) and (p,d) reactions. *Physical Review C*. 2011;84(3).
2. Alt EO, Blokhintsev LD, Mukhamedzhanov AM, Sattarov AI. Deuteron elastic scattering and stripping processes off ^{12}C as a three-body problem. *Physical Review C*. 2007;75(5).
3. Phillips A. Consistency of the low-energy three-nucleon observables and the separable interaction model. *Nuclear Physics A*. 1968;107(1):209-216.
4. Miyagawa K, Koike Y. Phenomenological Separable $\text{N-}^{12}\text{C}$ Interaction with Correct Energy-Dependence in Positive and Negative Energy Regions. *Progress of Theoretical Physics*. 1989;82(2):329-337.
5. Deltuva A. Spin observables in three-body direct nuclear reactions. *Nuclear Physics A*. 2009;821(1-4):72-79.
6. Machleidt R. High-precision, charge-dependent Bonn nucleon-nucleon potential. *Physical Review C*. 2001;63(2).
7. Watson BA, Singh PP, Segel RE. Optical-Model Analysis of Nucleon Scattering from 1p-Shell Nuclei between 10 and 50 MeV. *Physical Review*. 1969 ;182(4):977-989.
8. Menet JJH, Gross EE, Malanify JJ, Zucker A. Total-Reaction-Cross-Section Measurements for 30-60-MeV Protons and the Imaginary Optical Potential. *Physical Review C*. 1971;4(4):1114-1129.
9. Berthold GH, Stadler A, Zankel H. Faddeev calculations for low-energy p-d scattering. *Physical Review C*. 1990;41(4):1365-1383.
10. Lacombe M, Loiseau B, Richard JM, Mau RV, Côté J, Pirès P, de Tournelle R. Parametrization of the ParisN-N potential. *Physical Review C*. 1980;21(3):861-873.
11. Alt EO, Mukhamedzhanov AM, Nishonov MM, Sattarov AI. Proton-deuteron elastic scattering from 2.5 to 22.7 MeV. *Physical Review C*. 2002;65(6).
12. Experimental data taken from the National Nuclear Data Center, Brookhaven National Laboratory Online Data Service. <https://www.nndc.bnl.gov/exfor/>.