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PROGRESS IN THE MICROSCOPIC DESCRIPTION OF NUCLEON-NUCLEUS ELASTIC SCATTERING AT LOW ENERGY

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Abstract. In this brief report, we make a short review of progress in developing the microscopic optical potential in recent years. In particular, we present our current studies and plans on building the microscopic optical potential based on the so-called nuclear structure models at low energies.

Keywords: microscopic optical potential, effective Skyrme interaction, self-consistent mean-field

1 Introduction

In nuclear physics studies, we still have two unsolvable problems: the many-body problem and the nuclear interaction. To avoid these difficulties, we use phenomenological approaches as the most efficient way to describe nuclear systems, for example, the great success of using effective phenomenological interaction in nuclear structure and phenomenological optical potential in nuclear reactions. However, the limit of phenomenological approaches is the unexpected separation between the structure and reactions communities. Also, due to the fits with experimental data, these approaches do not have prediction powers, especially for the nuclear reactions off-targets outside the range of validity of the fits, e.g., the exotic nuclei produced in the r-process. However, the microscopic optical potential is expected to have prediction powers and a link between the nuclear structure and reactions studies. This link allows us to learn the physics meaning from analysing the experimental data from the studies of nuclear reactions.

In the last five years, tremendous efforts have been devoted to developing the microscopic optical potential. This potential is identified with the nucleon self-energy $\Sigma(r, r', E)$, which is a complex non-local energy-dependent function. When the incident energy $E > 0$, $\Sigma(r, r', E)$ is the nuclear optical potential. For bound states when $E < 0$, the real part of $\Sigma(r, r', E)$ represents the shell model or mean-field potential. We could list the most efficient methods to calculate this self-energy: *ab initio* approaches [1, 2], nuclear matter approaches [3-8], and nuclear structure approaches [9-19].

Recently, the combination of the Green's function approach with the couple-cluster method [2] has been used to generate the microscopic optical potential for neutron elastic scattering of ^{40}Ca and ^{48}Ca. This success is based on the progress of the *ab initio* nuclear reaction community in many aspects: mass number, precision, and accuracy. They can now have reliable predictions for nuclei as heavy as ^{120}Sn by using modern nucleon-nucleon (NN) and three-nucleon forces (3NFs) from the chiral effective field theory. In Ref. [2], the microscopic optical potential is defined as

$$\Sigma' \equiv \Sigma^* + U, \quad (1)$$

where $\Sigma^*(\gamma, \delta, E)$ is the self-energy, which can be calculated from

$$\Sigma^*(E) = [G^{0)}(E)]^{-1} - G^{-1}(E), \quad (2)$$

and U is the HF potential. The Green's function G fulfills the Dyson equation

$$G(\alpha, \beta, E) = G^{(0)}(\alpha, \beta, E) + \Sigma^{(0)}_{\gamma,\delta}(\alpha, \gamma, E) \times \Sigma^*(\gamma, \delta, E) G(\delta, \beta, E) \quad (3)$$

where $G^{(0)}$ is the first-order approximation to Green's function. The Dyson equation is obtained by inverting each element within the coupled-cluster method to get the optical potential. The method has been applied to describe the optical potential associated with the bound states in ^{41}Ca and ^{49}Ca and the neutron scattering. Some encouraging results are compared with the phenomenological Koning-Delaroche potential. However, because Eq. (2) is complicated, the imaginary part (the absorption from the non-elastic channel) of the optical potential is dropped.

Another way to calculate nucleon self-energy is to use the nuclear matter approach [3, 4]. Since this method is only valid for infinite nuclear matter, the local density approximation has been used. For getting the local optical potential, the solution of the self-consistent equation is folded with the resulting density-dependent mean-field with a realistic point-nucleus density distribution. In this approach, the first order of Σ is the Hartree-Fock contribution

$$\Sigma^{(1)}_{\text{HF}}(q, \omega; k_f) = \sum_l \langle \mathbf{q}\mathbf{h_1} s s_1 t t_1 | \tilde{V}_{2N} | \mathbf{q}\mathbf{h_1} s s_1 t t_1 \rangle n_1 \quad (4)$$

which is real, energy-independent, and $\tilde{V}_{2N}$ is the anti-symmetrization of V_{2N}.

The second order $\Sigma^{(1)}_{\text{HF}}(q, \omega; k_f)$ is both a real and imaginary part. The direct and exchange terms are calculated from the particle states above the Fermi level.

$$\Sigma^{(2a)}_{2\text{N}}(q, \omega; k_f) = \frac{1}{2} \sum_{123} \frac{|\langle \mathbf{p}_1 \mathbf{p}_3 s_1 s_3 t_1 t_3 | \tilde{V}_{2N} | \mathbf{q}\mathbf{h}_2 s s_2 t t_2 \rangle|^2}{\omega + \epsilon_2 - \epsilon_1 - \epsilon_3 + i\eta} \bar{n}_1 n_2 \bar{n}_3 (2\pi)^3 \delta(\mathbf{p}_1 + \mathbf{p}_3 - \mathbf{q} - \mathbf{h}_2) \quad (5)$$

where $\bar{n}_k = 1 - n_k$.

The microscopic optical potential calculations have been applied to study the real and imaginary part for incident energies lower than 100 MeV within the above formalism. However, no comparison with experimental data has been made. Also, it is well known that the local density approximation cannot capture the physics of the collective surface modes, shell structure effects, and the spin-orbit interaction. Later, several improvements have been proposed. The improved local density approximation has been used to consider the nonzero range of nuclear force. The calculation of the neutron elastic scattering off ^{40}Ca is performed at energies from 3.2 MeV up to 185 MeV. Good agreement with experimental data was obtained except in the resonances regions. However, the absorption of the imaginary part is still too large, and the spin-orbit interaction is also an intricate difficulty of the nuclear matter model.

Another way to calculate the microscopic optical potential (MOP) is to use the *ab initio* calculations based on the no-core-shell model by using saturating nuclear forces [1]. The Hamiltonian of the system is

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$$H(A) = \hat{T} - \hat{T}_{\text{c.m.}}(A+1) + \hat{V}_2 + \hat{V}_3 \quad (6)$$

where $\hat{T}$ is the kinetic energy operator, and $\hat{V}_2$, $\hat{V}_3$ are the two-body and three-body interactions. The partial wave decomposition of the self-energy, which is the optical potential, is

$$\Sigma^{l,j}(k,k';E,\Gamma) = \sum_{n,n'} R_{n,l}(k)\Sigma^{\star,l,j}_{n,n'}(E,\Gamma)R_{n',l}(k') \quad (7)$$

which is energy-dependent, nonlocal, and separated, and the irreducible self-energy $\Sigma^{\star}(\omega)$ is the solution of the Dyson equation

$$g(\omega) = g^0(\omega) + g^0(\omega)\Sigma^{\star}(\omega)g(\omega) \quad (8)$$

where $g^0(\omega)$ is the free particle propagator. This model has been applied to describe the neutron elastic scattering off ^{16}O (^{40}Ca) at 3.286 (3.2) MeV. The obtained results agree with experimental data. However, the model's main drawback is the impossibility of describing the critical collective states of the targets, such as low-lying states and giant resonances.

Based on the same Green function method, the nuclear structure model explicitly includes the effects of collective states, in which one uses two-body effective NN interactions together with some nuclear structure models to calculate the MOPs. As the recent energy-density-functional structure approaches have proven their ability to describe the nuclear structure observables in the stable region well, we can now generate the microscopic optical potential directly from the effective phenomenological interactions.

According to Refs. [10, 18-20], the MOPs are given as

$$V_{\text{opt}} = V_{\text{HF}} + \Delta\Sigma(\omega), \quad (9)$$

where

$$\Delta\Sigma(\omega) = \Sigma(\omega) - \frac{1}{2}\Sigma^{(2)}(\omega). \quad (10)$$

In Eqs. (9) and (10), V_{HF} is the real, local, momentum-dependent, energy-independent Skyrme HF mean-field potential, and ω is the nucleon incident energy. The polarization potential, $\Delta\Sigma(\omega)$, is non-local, complex, and energy-dependent. $\Sigma(\omega)$ is the contribution from the particle-hole correlations generated from fully self-consistent particle-vibration coupling (PVC) calculations [10, 18] applied on top of the collective states at small amplitudes generated by the Random Phase Approximation. The imaginary part of $\Sigma(\omega)$ is responsible for a loss of the incident flux due to the existence of nonelastic channels. $\Sigma^{(2)}(\omega)$ is the second-order potential generated from uncorrelated particle-hole contributions.

Using the partial wave expansion, we express the partial wave decomposition of the self-energy as

$$\Sigma_{lj}(r,r',\omega) = \hat{j}\sum_{\epsilon_\alpha,\epsilon_\beta} \frac{u_{lj}^{(\epsilon\alpha)}(r)}{r}\Sigma_{\alpha\beta}^{(lj)}(\omega)\frac{u_{lj}^{(\epsilon_\beta)}(r')}{r'}, \quad (11)$$

where $\hat{j} = (2j+1)^{1/2}$. The self-energy is non-local, energy-dependent, complex, and separated. These models have been used to reproduce the experimental data without *ad hoc* adjusted parameters on nucleon elastic scattering [10] by ^{208}Pb, neutron elastic scattering [13] by ^{16}O, proton inelastic scattering [14] by ^{24}O, nucleon elastic scattering by ^{40}Ca and ^{48}Ca [15-17], and nucleon elastic scattering [18, 19] by ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb in the framework of the nuclear structure model (NSM) (mainly with the Gogny and Skyrme interaction). However, the intricate disagreement with experimental data at backward angles does still exist.

Recently, we have used this model to analyse the effects of the spin-orbit and velocity-dependent terms on the nuclear reactions observables. The obtained results are relatively interesting. We have shown that the velocity-dependent terms have strong effects on the surface while the spin-orbit term contributes in the interior region [20]. We assume to evaluate the sensitivity of the angular distributions and analysing powers on each parameter of the Skyrme interactions in the near future. For example, Figure 1 depicts that the sensitivity of the angular distributions of the neutron elastic scattering at low energy depends on the interaction's choice. The obtained results show that this sensitivity is very small, between the SLy5 and SkM* effective interactions. More systematic calculations should be done in the near future. For the next step, the role of each of the multipolarities to the imaginary part will be analysed. This essential information will be used to build up the new generation of optical potential.

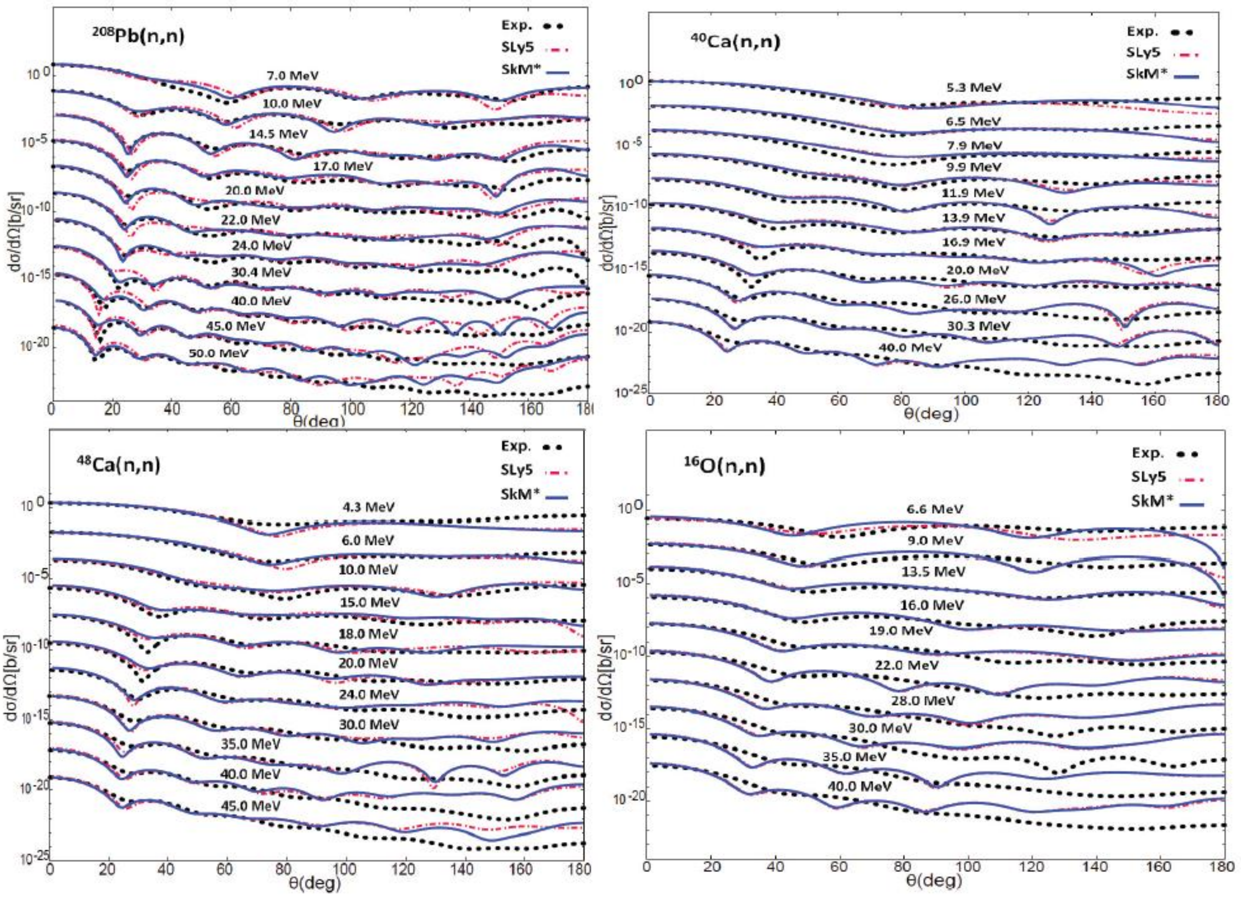

Fig. 1. Angular distributions of neutron elastic scattering by ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb at different incident energies below 50 MeV. The solid (dashed) curve shows the results of the MOP calculations using the SLy5 (SkM*) interaction. The results for SLy5 interaction are extracted from Ref. [19]. The experimental data are the tabulated cross sections taken from Ref. [21].

In conclusion, the microscopic optical potential is expected to be a vehicle to study nuclear reactions in unstable regions. However, the main challenge of this potential is its low precision compared with the phenomenological one, even in the stable region. As it is well known, this challenge results from the too complicated many-body problem underlying. Therefore, the combination of phenomenological and microscopic optical potential could be a promising research direction.

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