

Effects of velocity-dependent and spin-orbit terms of the Skyrme interaction on non-locality parameter of microscopic optical potentials

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Abstract. We investigate how the velocity-dependent (t_1 and t_2) and spin-orbit (W_0) terms of the Skyrme effective interaction influence the non-locality parameter β of the microscopic optical potential (MOP) for neutron elastic scattering off the doubly closed-shell nuclei ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at incident energies below 20 MeV. The MOP is constructed within the particle-vibration coupling framework on top of self-consistent Hartree-Fock plus random-phase approximation calculations by using the SLy5 parametrisation. Three calculation schemes are compared: the full Skyrme interaction, the interaction without t_1 and t_2 , and the interaction without W_0 in the residual interaction. The velocity-dependent terms exert a pronounced and mass-dependent influence on β : removing them increases β_v by 0.27 fm for ^{16}O but decreases it by 0.50 fm for ^{208}Pb . The spin-orbit term, by contrast, produces much weaker modifications, typically below 0.1 fm. The extracted β values lie in the range 0.70–1.43 fm, with most values exceeding the widely adopted Perey-Buck value of 0.85 fm, suggesting that this empirical parameter underestimates the true microscopic non-locality for most nuclei.

Keywords: non-locality parameter, microscopic optical potential, neutron elastic scattering, Skyrme interaction, velocity-dependent and spin-orbit terms

1 Introduction

The optical potential is one of the most fundamental quantities in nuclear reaction theory, providing a unified description of nucleon-nucleus elastic scattering, absorption, and the coupling among reaction channels [1–3]. A defining feature of this potential, when derived from microscopic many-body theory, is its inherent non-locality: the potential at a given point depends not only on the local coordinate but also on the wave function at neighbouring points. The physical origins of this non-locality are manifold, encompassing the antisymmetrisation of the many-body wave function demanded by the Pauli exclusion principle, the finite range of the underlying

nucleon-nucleon (N-N) interaction, channel-coupling effects, and contributions from multiple scatterings [2, 3].

In phenomenological treatments, non-locality is typically parameterised via the Perey-Buck prescription [4], which introduces a Gaussian non-locality kernel $\exp(-s^2/2\beta^2)$ with a single parameter β of 0.85 fm. This value was originally extracted from neutron elastic scattering data on ^{208}Pb at only two incident energies (7 MeV and 14.5 MeV). Despite this remarkably limited empirical constraint, the Perey-Buck potential has proven to be a surprisingly versatile tool, successfully describing nucleon-nucleus scattering across a broad range of targets and energies for

more than six decades. However, the limitations of the Perey-Buck approach have become increasingly apparent in modern applications. Titus et al. [5] demonstrated that the traditional Perey correction factor cannot fully account for the effects of non-locality, a finding corroborated by the work of Lovell and Nunes [6]. This shortcoming is particularly critical for transfer reactions such as (d, p) and (p, d) in inverse kinematics that serve as essential spectroscopic tools for studying exotic nuclei [7–10]. In such applications, a more reliable microscopic understanding of non-locality is required.

Microscopic optical potentials (MOPs) can be derived from a variety of many-body theoretical frameworks, including *ab initio* approaches [11, 12], nuclear matter calculations based on chiral effective field theory [13–15], semi-microscopic methods [16], the dispersive optical model [17, 18], and nuclear structure models [19–25]. At low incident energies (below 50 MeV), where specific nuclear-structure effects such as collective vibrational excitations become essential, nuclear-structure approaches based on self-consistent mean-field theories with effective phenomenological N-N interactions have proven especially valuable. Both the finite-range Gogny interaction [21–23] and the zero-range Skyrme interaction [24–26] were successfully employed within this framework. A comprehensive review of the current status of optical potentials for the rare-isotope beam era can be found in Ref. [27].

The pioneering microscopic study of non-locality was carried out by Vinh Mau and Bouyssy [28], who derived the imaginary part of the optical potential for neutron scattering on ^{40}Ca by using the Green function formalism incorporating random phase approximation (RPA) correlations. Employing a simple zero-range effective interaction without velocity-dependent or spin-dependent terms, they extracted a volume non-locality range of $\beta_v \approx 1.0$ fm. Bernard and Van Giai

[29] subsequently applied the particle-vibration coupling (PVC) approach with the Skyrme interaction to calculate the absorption potential for heavy nuclei, but with the spin-orbit, velocity-dependent, and spin-dependent terms omitted from the residual interaction because of computational limitations of that era. More recently, calculations with the finite-range Gogny interaction have been performed for ^{40}Ca and ^{48}Ca [21–23]. The non-local structure has also been investigated in momentum space by using the Brueckner-Hartree-Fock approximation, yielding β values between 0.86 and 0.89 fm for both proton and neutron scattering below 65 MeV [30, 31].

Building upon our earlier systematic study of the effects of the velocity-dependent and spin-orbit terms on scattering observables [26], the present work investigates how these specific terms of the Skyrme interaction influence the non-locality parameter β . The analysis covers both the volume and surface regions for ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at incident energies below 20 MeV, where the PVC framework provides the most reliable description, and experimental data are available for all four targets. This investigation is motivated by two observations. First, the velocity-dependent terms (t_1 and t_2) simulate the finite-range character of the N-N interaction within the zero-range Skyrme framework, and their contribution to non-locality is therefore expected to be significant. Second, the spin-orbit term (W_0) is essential for describing spin observables, but its role in determining the scalar non-locality remains poorly understood. Clarifying the individual contributions of these interaction terms to the non-locality is crucial for developing reliable optical potentials for exotic nuclei where experimental constraints are severely limited.

2 Theoretical framework

2.1 The Skyrme effective interaction

The standard form of the Skyrme effective interaction between two nucleons is given by

$$\begin{aligned} V_{\text{Skyrme}} = & t_0(1 + x_0 P^\sigma)\delta(\mathbf{r}) \\ & + (1/2)t_1(1 + x_1 P^\sigma)[\mathbf{k}'^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{k}^2] \\ & + t_2(1 + x_2 P^\sigma) \mathbf{k}' \cdot \delta(\mathbf{r}) \mathbf{k} \\ & + iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot [\mathbf{k}' \times \delta(\mathbf{r}) \mathbf{k}] \\ & + (1/6)t_3(1 + x_3 P^\sigma) \rho^a(\mathbf{R}) \delta(\mathbf{r}) \end{aligned} \quad (1)$$

[32, 33], where r ($r_1 - r_2$) is the relative coordinate; R ($(1/2)(r_1 + r_2)$) is the centre-of-mass coordinate; k ($(1/2i)(\nabla_1 - \nabla_2)$) is the relative momentum operator acting to the right; k' is its Hermitian conjugate acting to the left. The spin-exchange operator is $P^\sigma = (1/2)(1 + \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)$; ρ denotes the total nucleon density. The parameters $t_0, t_1, t_2, t_3, W_0, \alpha, x_0, x_1, x_2$, and x_3 are determined by fitting to experimental data [33].

The five terms in Eq. (1) have distinct physical roles. The t_0 term provides the central, zero-range part of the interaction. The terms proportional to t_1 and t_2 are the velocity-dependent terms that account for the momentum dependence of the effective interaction and effectively simulate the finite-range character of the bare N-N force within the zero-range framework [26]. The term proportional to W_0 is the two-body spin-orbit interaction, responsible for the spin-orbit splitting of single-particle levels and plays a crucial role in describing spin-dependent observables, such as analysing powers [34, 35]. Finally, the t_3 term is the density-dependent term that accounts for many-body effects beyond the two-body level.

2.2 The microscopic optical potential

The microscopic optical potential is constructed following the PVC framework developed in Refs. [24–26]

$$V_{\text{opt}} = V_{\text{HF}} + \Delta\Sigma(\omega) \quad (2)$$

where V_{HF} is the static Skyrme-Hartree-Fock (HF) mean field, which is real, local, and energy-independent. It provides the dominant contribution to the real part of the optical potential. The locality of V_{HF} is a direct consequence of adopting the zero-range Skyrme interaction. The dynamical part is given by

$$\Delta\Sigma(\omega) = \Sigma(\omega) - (1/2) \Sigma^{(2)}(\omega) \quad (3)$$

where ω is the nucleon incident energy. The first-order self-energy $\Sigma(\omega)$ arises from the coupling of single-particle states to collective phonon excitations built from particle-hole correlations within the RPA framework. The second-order potential $\Sigma^{(2)}(\omega)$ is included to correct for the violation of the Pauli principle in the particle-vibration coupling [24, 29].

The dynamical potential $\Delta\Sigma(\omega)$ is non-local, complex, and energy-dependent. Its imaginary part is responsible for the absorption in the optical potential, describing the loss of flux from the elastic channel to various non-elastic channels. By virtue of the partial-wave expansion with imposed spherical symmetry, the (l, j) components of $\Sigma(\omega)$ read as

$$\begin{aligned} \Sigma_{\alpha\beta}^{(lj)}(\omega) = & \hat{j}_\alpha^{-1} \hat{j}_\beta^{-1} \left(\sum_{nL, A > F} \frac{\langle \alpha \| V \| A, nL \rangle \langle A, nL \| V \| \beta \rangle}{\omega - \epsilon_A - \omega_{nL} + i\eta} \right. \\ & \left. + \sum_{nL, a < F} \frac{\langle \alpha \| V \| a, nL \rangle \langle a, nL \| V \| \beta \rangle}{\omega - \epsilon_a + \omega_{nL} - i\eta} \right), \end{aligned} \quad (4)$$

[24, 25], where α and β are generic single-particle states; $a(A)$ denotes the hole (particle) single-particle states; ϵ_j are the single-particle HF energies; ω_{nL} are the n th phonon energies with multipolarity L ; $\tilde{j} = (2j+1)^{1/2}$; F denotes the Fermi level. The parameter η ($= 1.5$ MeV) is introduced to perform energy averaging on the potential $\Delta\Sigma(\omega)$. The reduced matrix elements $\langle i \| V \| j, nL \rangle$ are the particle-vibration couplings, calculated as described in Refs. [26, 34, 35]. In the present fully self-consistent approach, the residual interaction V

is consistently treated by using the full Skyrme interaction, including all velocity-dependent and spin-orbit terms.

2.3 Characterising non-locality

To quantify the non-locality of the imaginary part of the optical potential, we follow the approach originally introduced by Vinh Mau and Bouyssy [28]. We define the quantity

$$W(R, s, \omega) = \Sigma_{lj} (2j+1)/(4\pi) \text{Im} \Delta \Sigma_{lj}(r, r', \omega), \quad (5)$$

where $R = ((1/2)(r + r'))$ is the centre-of-mass coordinate characterising the radial position within the nucleus; $s = (r - r')$ is the non-locality variable measuring the spatial extent of the off-diagonal behaviour of the potential; ω is the incident energy. As demonstrated in Ref. [28], $W(R, s, \omega)$ depends only on the magnitudes R and s and exhibits a Gaussian dependence on s for all values of R and incident energy.

The non-locality parameter β is extracted by fitting the s -dependence to the Perey-Buck Gaussian form, as follows:

$$W(R, s) = W(R, 0) \times \exp(-s^2 / 2\beta^2), \quad (6)$$

where β may depend on R . In particular, we distinguish between the volume non-locality parameter β_v , extracted in the nuclear interior region, and the surface non-locality parameter β_s , extracted at the nuclear surface where absorption is strongest.

2.4 Computational details

All calculations are performed via the SLy5 parametrisation of the Skyrme effective interaction [33]. The radial HF equations are solved in coordinate space with a mesh size of 0.1 fm and a maximum radial extent of 15 fm. Spherical symmetry is imposed throughout, which is appropriate for the doubly closed-shell target nuclei ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb considered in this work.

Ground states and excited states are constructed on top of the HF solution by using fully self-consistent RPA calculations [36]. All natural-parity RPA excited states with multipolarities $L = 0^+, 1^-, 2^+, 3^-, 4^+$, and 5^- and excitation energies below 50 MeV are included in the PVC calculations. Continuum states with positive energy are treated by discretising the spectrum under box boundary conditions. Unnatural-parity states have been excluded from the present analysis. All computational parameters are kept fixed for all nuclei and at all excitation energies considered.

To disentangle the individual contributions of the velocity-dependent and spin-orbit terms to the non-locality, we perform three distinct sets of calculations:

- (i) **Full**: The complete Skyrme interaction is used in all stages of the calculation.
- (ii) **No t_1, t_2** : The velocity-dependent terms (t_1 and t_2) are removed from the residual interaction entering the PVC calculation.
- (iii) **No spin-orbit**: The spin-orbit term (W_0) is removed from the residual interaction in the PVC calculation.

In all three cases, the HF mean field is calculated with the full Skyrme interaction, ensuring that the single-particle spectrum and ground-state properties remain identical. This procedure isolates the effect of each interaction term on the dynamical part of the optical potential and, consequently, on the non-locality parameter. We emphasise that this approach is intentional rather than approximate. Our objective is to disentangle the role played by the t_1, t_2 and W_0 terms within the dynamical PVC self-energy specifically, while keeping the underlying nuclear structure (single-particle energies, wave functions, and RPA phonon spectra) exactly the same in all the three schemes. An alternative scheme in which the t_1 and t_2 terms are also removed at the HF level would simultaneously alter the effective mass

m^*/m , the single-particle level spacing, and the entire RPA spectrum; the resulting changes in β would then mix true PVC-vertex effects with structural modifications and would no longer permit a clean attribution to the velocity-dependent character of the residual interaction. The present scheme should therefore be interpreted as a controlled sensitivity analysis of the PVC vertex, not as a globally consistent “no- t_1, t_2 ” calculation. Comparison with the fully consistent zero-range calculation of Vinh Mau and Bouyssy [28], who excluded velocity-dependent terms throughout, will be discussed in Sec. 3.5 and shown to corroborate the present interpretation.

The nuclear reaction observables are obtained by solving the Schrödinger equation with the standard DWBA98 code [37] by using the non-local, complex, energy-dependent MOP of Eq. (2). Experimental data are taken from the National Nuclear Data Center [38].

The non-locality parameter β is extracted by performing Gaussian fits according to Eq. (6) at two characteristic radial positions for each target nucleus: a volume radius R_v in the nuclear interior where the density is approximately uniform, and a surface radius R_s near the nuclear surface where the absorption reaches its maximum value. The volume radius R_v and surface radius R_s are determined by examining the radial dependence of $W(R, s)$ at fixed values of s : R_v is chosen at the position of the local maximum of $|W(R, s = 0)|$ in the nuclear interior (when present), and R_s at the position of the absolute maximum corresponding to the surface peak of the absorption. We verified that these positions remain essentially unchanged across the range of incident energies considered for each target nucleus. The specific values used in this work are as follows: $R_v = 0.8$ fm and $R_s = 3.7$ fm for ^{16}O ; $R_v = 2.8$ fm and $R_s = 4.7$ fm for ^{40}Ca ; $R_v = 3.0$ fm and $R_s = 4.9$ fm for ^{48}Ca ; and $R_v = 5.7$ fm and $R_s = 7.6$ fm for ^{208}Pb . We note that $R_v = 0.8$ fm for ^{16}O corresponds to the local maximum of $|W(R, s = 0)|$

in the interior of this light nucleus, where the density is approximately constant. For ^{208}Pb , $R_v (= 5.7$ fm) lies well inside the half-density radius ($R_{1/2} \approx 6.7$ fm) and corresponds to a well-defined volume absorption peak visible in $W(R, s = 0)$. We verified that moderate variations of R_v and R_s by ± 0.3 fm around the chosen values produce changes in β of at most ± 0.03 fm, confirming that our results are not sensitive to the precise choice of evaluation radii. We also checked that the extracted β values are stable under variations in the energy-averaging parameter η within the range 1.0–2.0 MeV, with resulting changes in β remaining below 0.02 fm. The fitting range in the non-locality variable s is set to $s_{\text{max}} = 4.0$ fm for all surface fits and for the volume fits of ^{16}O and ^{40}Ca , while a reduced range of $s_{\text{max}} = 2.0$ fm is employed for the volume fits of ^{48}Ca and ^{208}Pb . This restriction is necessary because, for heavier nuclei at small radii, the imaginary potential $W(R_v, s)$ develops oscillatory structures beyond $s \approx 2$ fm that arise from the interference between different PVC contributions. These structures cause the Gaussian fit quality $\mathcal{R}^2$ to degrade significantly when the full s range is used; restricting to $s \leq 2.0$ fm restores excellent fit quality ($\mathcal{R}^2 > 0.95$ in most cases). We emphasise that the restricted-range β_v remains physically meaningful as a measure of non-locality rather than merely reflecting a local feature of $W(R_v, s)$. The Perey-Buck Gaussian kernel decays from its central value when $s = 0$ to $\exp(-2/\beta^2)$ when $s = 2.0$ fm, which corresponds to roughly 14–36% of $W(R_v, 0)$ for the typical $\beta_v \sim 1$ –1.4 fm encountered in the volume region of ^{48}Ca and ^{208}Pb . The dominant Gaussian core is therefore within $s \leq 2.0$ fm, while the oscillatory structures at larger s contribute only a modest additional weight to the kernel. The restricted fit captures this dominant Gaussian core; the residual oscillations when $s > 2$ fm represent finer structure in the PVC self-energy that lies outside the Perey-Buck ansatz and would in any case not be reproducible by a single β parameter.

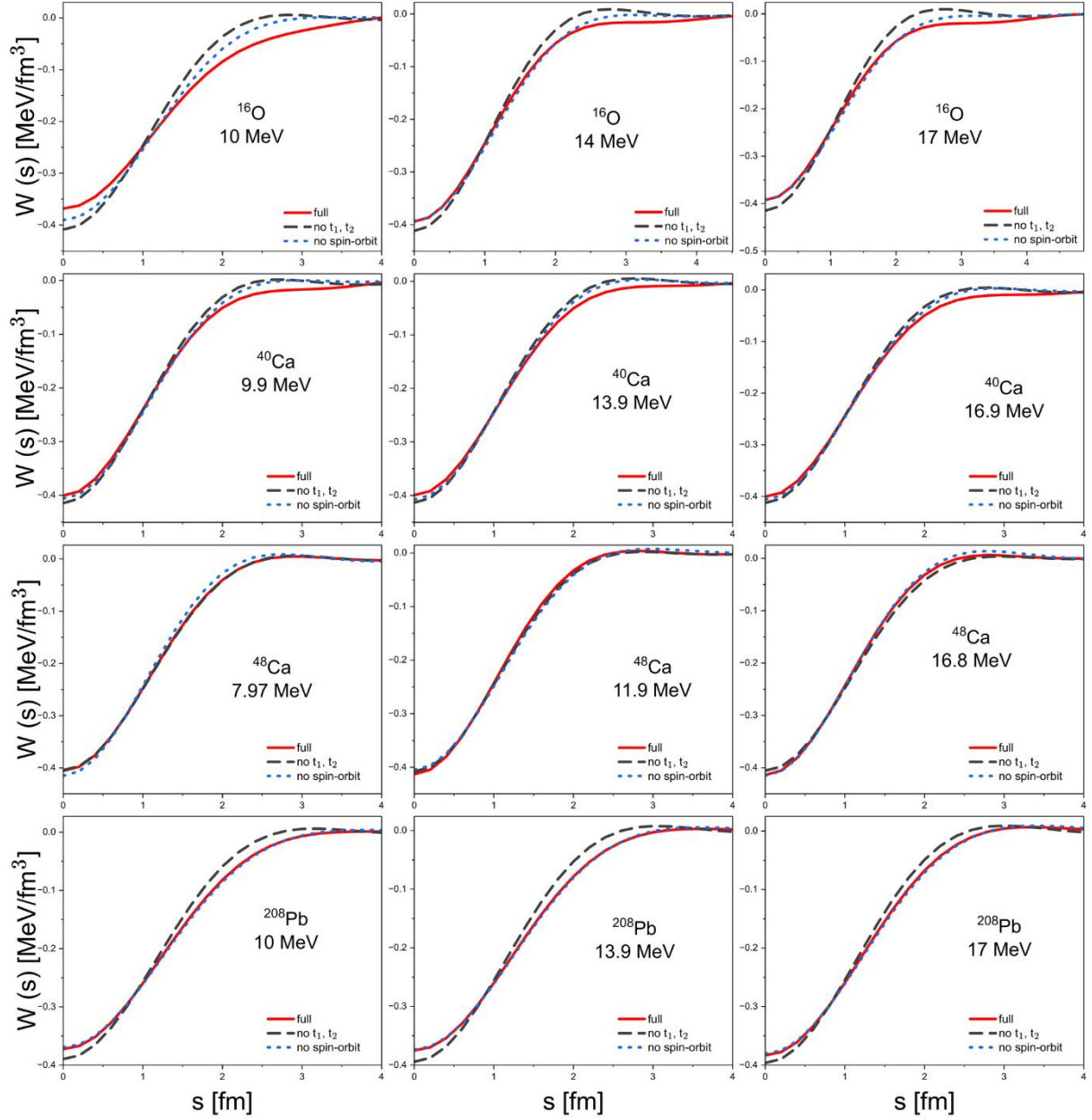


Fig. 1. Imaginary part of non-local optical potential $W(R_s, s)$ evaluated at surface radius R_s as function of non-locality variable s , for neutron elastic scattering off ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at three incident energies (rows) and in three calculation schemes: full Skyrme interaction (red solid), without t_1 and t_2 (black dashed), and without spin-orbit (blue dashed)

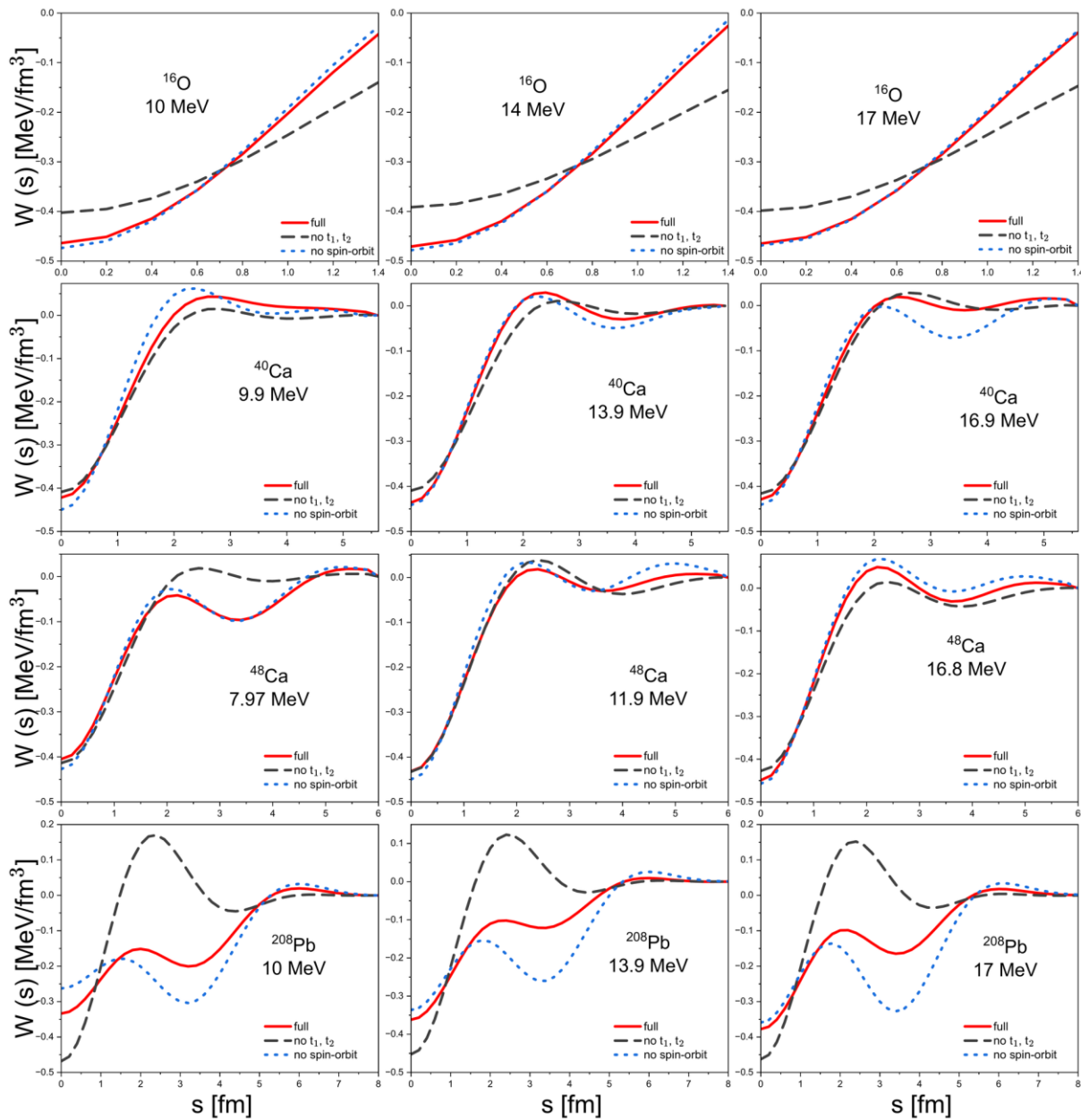


Fig. 2. Same as Fig. 1 but evaluated at volume radius R_v

3 Results and discussion

3.1 Non-local structure of the imaginary potential

Figs. 1 and 2 display the imaginary part $W(R, s)$ as a function of the non-locality variable s at the surface and volume radii, respectively, for all four target nuclei at different incident energies. The results from all three calculation schemes are

presented. Several key features emerge from this analysis.

First, for all nuclei and incident energies, $W(R, s)$ exhibits a clear Gaussian shape as a function of s at the nuclear surface, confirming the validity of the Perey-Buck ansatz [Eq. (6)] and justifying the extraction of β from a Gaussian fit. This observation is consistent with previous findings from Gogny-based [21] calculations and from momentum-space studies [30].

Second, the width of the Gaussian, and hence the non-locality parameter β , varies between the volume and surface regions. For the light nucleus ^{16}O , the surface non-locality is significantly broader ($\beta_s > \beta_v$), reflecting the different physical mechanisms governing absorption in the nuclear interior versus the surface. For medium-mass nuclei (^{40}Ca and ^{48}Ca), the difference between β_v and β_s becomes smaller.

Third, at the volume radius of heavier nuclei, particularly ^{48}Ca and ^{208}Pb , the potential $W(R_v, s)$ develops oscillatory structures at larger values of s that deviate from the simple Gaussian form. These structures arise from the interference among different PVC contributions and are most pronounced for ^{208}Pb , where the no-spin-orbit calculation shows especially strong oscillations. As discussed in Sec. 2.4, a restricted fitting range ($s_{\text{max}} = 2.0$ fm) is employed in these cases to ensure reliable extraction of β_v .

Fourth, removing the velocity-dependent terms produces a clearly visible change in the Gaussian width, particularly in the volume region of light nuclei. Removing the spin-orbit term, by contrast, produces only subtle modifications to the non-local structure.

3.2 Extracted non-locality parameters

Table 1 summarises the non-locality parameters β_v and β_s together with the Gaussian fit quality $\mathcal{R}^2$

extracted from Gaussian fits to $W(R, s)$ for all nuclei and incident energies in the three calculation schemes. The coefficient of determination $\mathcal{R}^2$ measures how well the Gaussian ansatz of Eq. (6) reproduces the computed $W(R, s)$ profile; values closer to unity indicate better agreement. For most entries, the fit quality $\mathcal{R}^2$ exceeds 0.92 at the volume radius and 0.99 at the surface, indicating that the Perey-Buck Gaussian form provides an excellent representation of $W(R, s)$. A small number of entries with lower $\mathcal{R}^2_v$, including the case of ^{208}Pb without spin-orbit when $E = 10$ MeV, which is marked as ‘–’ in the table, are discussed in detail below.

The quoted β values reflect an intrinsic fitting uncertainty that we estimate as follows. The standard errors resulting from the Levenberg-Marquardt least-squares procedure are typically below ± 0.01 fm for fits where $\mathcal{R}^2 > 0.95$, and up to ± 0.03 fm for the cases with lower $\mathcal{R}^2$. Combining the ± 0.03 fm sensitivity with the choice of evaluation radii (Sec. 2.4), we estimate an overall uncertainty of approximately ± 0.03 – 0.04 fm for entries with $\mathcal{R}^2 \geq 0.95$, and up to ± 0.06 fm for entries where $\mathcal{R}^2 < 0.90$. The differences, $\Delta\beta$, discussed in the following sections are generally well above these uncertainty estimates, except for the spin-orbit effects that have comparable magnitudes in light nuclei.

Table 1. Non-locality parameters β (in fm) and Gaussian fit quality $\mathcal{R}^2$ for neutron elastic scattering off ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at selected incident energies

Nucleus	E (MeV)	Full				No t_1, t_2				No spin-orbit			
		β_v	$\mathcal{R}^2_v$	β_s	$\mathcal{R}^2_s$	β_v	$\mathcal{R}^2_v$	β_s	$\mathcal{R}^2_s$	β_v	$\mathcal{R}^2_v$	β_s	$\mathcal{R}^2_s$
^{16}O	10.0	0.74	0.99	1.17	0.99	0.98	0.99	0.95	0.99	0.71	0.98	1.04	0.99
	14.0	0.72	0.98	1.02	0.99	1.04	0.99	0.94	0.99	0.70	0.98	1.03	0.99
	17.0	0.74	0.99	1.03	0.99	0.99	0.99	0.92	0.99	0.72	0.99	1.04	0.99
^{40}Ca	9.9	0.90	0.99	0.99	0.99	0.95	0.99	0.93	0.99	0.79	0.99	0.97	0.99

Nucleus	E (MeV)	Full				No t_1, t_2				No spin-orbit			
		β_v	$\mathcal{R}^2_v$	β_s	$\mathcal{R}^2_s$	β_v	$\mathcal{R}^2_v$	β_s	$\mathcal{R}^2_s$	β_v	$\mathcal{R}^2_v$	β_s	$\mathcal{R}^2_s$
	13.9	0.84	0.99	0.99	0.99	0.95	0.99	0.93	0.99	0.82	0.97	0.96	0.99
	16.9	0.86	0.99	0.99	0.99	0.92	0.99	0.94	0.99	0.82	0.94	0.96	0.99
	7.97	0.97 ⁺	0.83	0.97	0.99	0.93 ⁺	0.99	0.97	0.99	0.87 ⁺	0.85	0.92	0.99
⁴⁸ Ca	11.9	0.85 ⁺	0.99	0.93	0.99	0.85 ⁺	0.98	0.96	0.99	0.79 ⁺	0.99	0.97	0.99
	16.8	0.79 ⁺	0.99	0.93	0.99	0.87 ⁺	0.98	0.97	0.99	0.76 ⁺	0.99	0.93	0.99
	10.0	1.43 ⁺	0.96	1.15	0.99	0.73 ⁺	0.98	1.05	0.99	– ⁺	0.64	1.16	0.99
²⁰⁸ Pb	13.9	1.22 ⁺	0.99	1.13	0.99	0.78 ⁺	0.98	1.02	0.99	1.41 ⁺	0.92	1.14	0.99
	17.0	1.12 ⁺	0.99	1.08	0.99	0.74 ⁺	0.98	1.01	0.99	1.23 ⁺	0.92	1.10	0.99

Notes: β_v and β_s denote the values extracted at the volume and surface radii, respectively. The coefficient of determination $\mathcal{R}^2$ quantifies how well the Gaussian form of Eq. (6) reproduces the calculated $W(R, s)$; values closer to unity indicate better agreement. The calligraphic symbol $\mathcal{R}$ is used to distinguish the fit quality metric from the nuclear radius R . The fitting range is $s_{\text{max}} = 4.0$ fm except for ⁴⁸Ca and ²⁰⁸Pb at the volume radius where $s_{\text{max}} = 2.0$ fm is used (marked by ⁺). For ²⁰⁸Pb without spin-orbit when $E = 10$ MeV, the fit quality is too poor ($\mathcal{R}^2_v = 0.64$) for reliable β_v extraction (marked ‘–’). A few entries in Table 1 require a more detailed discussion of the Gaussian-fit reliability. The lowest values of $\mathcal{R}^2_v$ in the volume region appear for ⁴⁸Ca when $E = 7.97$ MeV in the full ($\mathcal{R}^2_v = 0.83$) and ‘no spin-orbit’ ($\mathcal{R}^2_v = 0.85$) schemes, and for ⁴⁰Ca without spin-orbit when $E = 16.9$ MeV ($\mathcal{R}^2_v = 0.94$). Inspection of the corresponding curves in Fig. 2 shows that these cases retain an overall Gaussian core but exhibit deviations from the simple Gaussian form at intermediate s that the single-parameter Perey-Buck ansatz cannot reproduce; the resulting β_v should therefore be interpreted as the effective Gaussian width that best represents the central core of $W(R_v, s)$, with the understanding that the genuine non-local structure has a non-negligible non-Gaussian component in these specific cases. It is noteworthy, however, that the corresponding ‘No t_1, t_2 ’ calculations at the same nucleus and energy yield much higher fit qualities ($\mathcal{R}^2_v = 0.99$ in all the three cases), indicating that the deviations from the Gaussian form are tightly linked to the presence of the velocity-dependent terms in the residual interaction and are therefore physical rather than numerical in origin. The most extreme case is ²⁰⁸Pb without spin-orbit when $E = 10$ MeV ($\mathcal{R}^2_v = 0.64$), where the oscillatory structure dominates over the Gaussian core; we mark this entry by ‘–’ as no single β_v can faithfully characterise the kernel. For the ²⁰⁸Pb ‘no spin-orbit’ entries at higher energies ($\mathcal{R}^2_v = 0.92$), the corresponding β_v values (1.41 and 1.23 fm) should be considered indicative rather than precise. With the single exception of ⁴⁸Ca when $E = 7.97$ MeV mentioned above, all other ‘full’ calculations yield $\mathcal{R}^2_v \geq 0.96$ and provide reliable benchmarks for assessing the effects of the residual interaction terms.

3.3 Effects of the velocity-dependent terms

The removal of the t_1 and t_2 terms from the residual interaction produces significant changes in the non-locality parameter, with a pronounced mass dependence. The results reveal a rich and physically informative pattern.

For the light nucleus ¹⁶O, removing the velocity-dependent terms increases β_v by

approximately 0.27 fm on average (from ~0.73 fm to ~1.01 fm), while simultaneously decreasing β_s by about 0.14 fm (from ~1.08 fm to ~0.94 fm). This substantial effect indicates that the velocity-dependent terms act to narrow the non-locality kernel in the nuclear interior while broadening it at the surface. The underlying mechanism can be understood as follows: the t_1 and t_2 terms simulate the finite-range character of the N-N interaction [26], and their presence in the PVC vertex modifies

the coupling strengths to phonons in a way that is spatially dependent. In the nuclear interior, where the density is nearly uniform, the velocity-dependent terms tend to reduce the coupling strength to phonons [26], leading to a more compact non-locality kernel. At the surface, the opposite tendency is observed, reflecting the different density gradients and single-particle wave function profiles.

For the medium-mass nucleus ^{40}Ca , the effect is smaller but still appreciable: $\Delta\beta_v \approx +0.07$ fm and $\Delta\beta_s \approx -0.06$ fm. This finding is in excellent agreement with the pioneering calculation of Vinh Mau and Bouyssy [28], who obtained a β_v value of 1.0 fm for ^{40}Ca using a simple zero-range interaction without velocity-dependent terms. Our corresponding ‘No t_1, t_2 ’ calculation yields $\beta_v \approx 0.94$ fm, consistent with their classic result within the modern fully self-consistent framework.

The behaviour of ^{48}Ca is qualitatively different. The volume parameter changes only slightly ($\Delta\beta_v \approx +0.01$ fm on average), while the surface parameter increases ($\Delta\beta_s \approx +0.02$ fm). Compared with ^{40}Ca , this distinct response can be attributed to the neutron excess in ^{48}Ca ($N - Z = 8$), which modifies the density profile and the structure of the collective excitations.

Most strikingly, for the heavy nucleus ^{208}Pb , the trend reverses completely: β_v decreases dramatically when the velocity-dependent terms are removed, from ~ 1.25 fm to ~ 0.75 fm (a change of ≈ -0.50 fm). This mass-dependent reversal is a non-trivial prediction of the fully self-consistent calculation and reflects the interplay between the velocity-dependent terms and the nuclear density profile, which varies dramatically from light (^{16}O , central density ~ 0.14 fm $^{-3}$) to heavy (^{208}Pb , central density ~ 0.16 fm $^{-3}$) systems. The physical origin of this reversal can be traced to the structure of the PVC coupling vertex: in ^{208}Pb , the large number of particle-hole configurations and the dense spectrum of collective phonon states, particularly

high-multipolarity modes ($L = 3^-, 4^+$, and 5^-) that carry significant strength in the nuclear interior, produce extended spatial correlations in the self-energy when mediated by the momentum-dependent t_1 and t_2 vertex factors. Removing these factors suppresses the long-range coherence among different phonon contributions, thereby reducing β_v . By contrast, for light nuclei such as ^{16}O where only a few low-lying collective modes dominate, and the volume region is small, the t_1 and t_2 terms instead act to compress the non-locality kernel by enhancing Pauli blocking effects in the nuclear interior.

To put the reversal on a more quantitative footing, it is useful to compare the absolute magnitudes of β_v in the two ‘No t_1, t_2 ’ schemes across nuclei. The averaged ‘No t_1, t_2 ’ values (β_v) are ≈ 1.00 fm for ^{16}O , 0.94 fm for ^{40}Ca , 0.88 fm for ^{48}Ca , and 0.75 fm for ^{208}Pb . They span a relatively narrow range of 0.75–1.00 fm, in contrast to the much wider range 0.73–1.43 fm spanned by the corresponding ‘Full’ calculations. Furthermore, the ^{40}Ca value sits close to the long-standing $\beta_v \sim 1$ fm benchmark of Vinh Mau and Bouyssy [28]. The reversal is therefore not a feature of the ‘No t_1, t_2 ’ calculation, where the mass dependence of β_v is mild, but a feature of the full calculation, where the t_1 and t_2 vertex amplifies the volume non-locality in heavy systems while compressing it in light systems. Quantitatively, the ratio $\beta_v^{\text{full}}/\beta_v^{\text{not } t_1 t_2}$ averaged over the considered energies varies monotonically with mass: 0.73 for ^{16}O , 0.92 for ^{40}Ca , 0.98 for ^{48}Ca , and 1.68 for ^{208}Pb . This trend supports the interpretation in terms of an interplay between the spatial range of the velocity-dependent vertex and the size of the volume region: for ^{16}O , the t_1 and t_2 vertices compress β_v ; for ^{40}Ca and ^{48}Ca , the effect is essentially neutral ($\beta_v^{\text{full}} \approx \beta_v^{\text{not } t_1 t_2}$ within $\sim 10\%$), and for ^{208}Pb , it strongly amplifies β_v . The transition from compression to amplification therefore occurs gradually between $A \sim 40$ and $A \sim 200$.

These results collectively demonstrate that the velocity-dependent terms play a crucial and mass-dependent role in determining the non-local structure of the microscopic optical potential. Any attempt to parameterise non-locality with a single, universal value of β is therefore an oversimplification.

3.4 Effects of the spin-orbit interaction

In marked contrast to the velocity-dependent terms, removing the spin-orbit interaction from the PVC calculation produces only modest changes in the non-locality parameter. For most nuclei and energies considered, the changes $|\Delta\beta|$ are typically less than 0.1 fm. This result is physically consistent with the observation that the spin-orbit interaction primarily affects spin-dependent observables such as analysing powers [26], while its contribution to the scalar (spin-averaged) non-locality is relatively weak.

Examining the results in more detail, we note that the largest spin-orbit effects on β_v appear for ^{48}Ca and ^{40}Ca , where removing the spin-orbit term decreases β_v by approximately 0.06–0.07 fm on average. For ^{16}O , the changes in β_v remain below 0.05 fm at all energies considered, while for ^{48}Ca , they reach 0.07–0.10 fm at the lower energies and decrease to about 0.03 fm at the highest energy. These variations show no simple systematic trend with incident energy.

The case of ^{208}Pb deserves special attention. When $E = 10$ MeV, the removal of the spin-orbit term produces strongly oscillatory behaviour in $W(R_v, s)$ that severely degrades the Gaussian fit quality ($\mathcal{R}^2_v = 0.64$), rendering the extracted β_v unreliable; this case is therefore excluded from Table 1. At higher energies (13.9 and 17 MeV), the fit quality improves but remains lower than that in the full calculation. This indicates that the spin-orbit contribution to the PVC vertex is crucial for maintaining the Gaussian character of $W(R_v, s)$ in

the interior of ^{208}Pb . Without it, strong oscillatory interference patterns emerge, preventing meaningful extraction of a single β_v . At the surface, by contrast, the spin-orbit removal has a weak effect ($\Delta\beta_s < 0.05$ fm) for all nuclei, including ^{208}Pb , which is consistent with the surface region being less sensitive to the details of the residual interaction.

This finding has important practical implications. For the calculations focused primarily on extracting non-locality parameters in light and medium-mass nuclei, the computationally expensive spin-orbit contribution to the PVC vertex can be omitted without significantly compromising the accuracy of β . This is particularly relevant, given that including the spin-orbit term in the PVC calculation increases the computational cost by approximately a factor of ten [26]. However, for heavy nuclei such as ^{208}Pb , the spin-orbit term is essential for obtaining reliable volume non-locality parameters.

3.5 Comparison with previous calculations and the Perey-Buck value

Our results permit a comprehensive comparison with previous theoretical studies of non-locality.

The comparison with the pioneering work of Vinh Mau and Bouyssy [28] on ^{40}Ca is particularly instructive. In their seminal study, the imaginary part of the optical potential was derived within the Green function formalism with RPA correlations, employing a simple zero-range effective interaction without velocity-dependent or spin-dependent terms. From the Gaussian shape of $W(R, s)$, they extracted a volume non-locality range of $\beta_v \approx 1.0$ fm, which has remained one of the few microscopic estimates of this quantity for decades. Our corresponding ‘No t_1, t_2 ’ calculation for ^{40}Ca yields $\beta_v \approx 0.94$ fm averaged over the three incident energies considered, which is in excellent agreement with their result. This agreement is remarkable, considering the major methodological

differences between the two approaches: Vinh Mau and Bouyssy [28] used a phenomenological zero-range force without self-consistency, whereas the present calculation is based on the modern SLy5 Skyrme parametrisation within a fully self-consistent HF-RPA-PVC framework with a much richer set of collective excitations. The close agreement thus provides strong evidence that the volume non-locality range is largely determined by the zero-range character of the underlying interaction rather than by the specific details of the residual interaction or the model space, confirming the robustness of the $\beta_v \sim 1$ fm estimate in the absence of velocity-dependent terms.

The comparison with the work of Bernard and Van Giai [29] on ^{208}Pb is particularly important, as it represents the first application of the PVC approach with the Skyrme interaction to a heavy nucleus. Using the SIII parametrisation and omitting the velocity-dependent, spin-orbit, and spin-dependent terms from the residual interaction due to the computational limitations of that era, they obtained surface non-locality values $\beta_s = 0.92\text{--}0.98$ fm at incident energies up to 30 MeV. Our full calculation yields somewhat larger surface values of $\beta_s \approx 1.08\text{--}1.15$ fm for the same nucleus at comparable energies. This $\sim 17\%$ enhancement can be attributed to several interrelated factors: (i) the inclusion of all natural-parity RPA multipolarities up to $L = 5^-$ in the present work, compared with the more restricted phonon space accessible in Ref. [29]; (ii) the fully self-consistent treatment of the residual interaction, which ensures that the same Skyrme force is used at the HF, RPA, and PVC levels; (iii) the Pauli correction through the second-order potential $\Sigma^{(2)}(\omega)$, which was not included in the original Bernard-Van Giai calculation; (iv) the use of the modern SLy5 parametrisation, which better describes the nuclear surface properties relevant for the absorption mechanism. More importantly, whereas Bernard and Van Giai [29] could not extract a volume non-locality parameter

for ^{208}Pb because of the vanishing absorptive strength in the nuclear interior of their calculation, our fully self-consistent approach reveals a well-defined volume absorption with β_v reaching values as large as 1.43 fm at low energies. This substantial volume non-locality in the heaviest doubly-magic nucleus represents a qualitatively new finding that was inaccessible to the original Skyrme-based PVC calculations.

Turning from microscopic to phenomenological benchmarks, we can see that our results are comparable with the Perey-Buck value of $\beta = 0.85$ fm. Our full calculations yield β_v values ranging from 0.72 fm (^{16}O at 14 MeV) to 1.43 fm (^{208}Pb at 10 MeV), while β_s ranges from 0.93 fm (^{48}Ca at higher energies) to 1.17 fm (^{16}O at 10 MeV). While the volume region of ^{16}O consistently gives $\beta_v < 0.85$ fm, the majority of extracted β values, particularly at the nuclear surface and for medium-to-heavy nuclei, are larger than the Perey-Buck value. This suggests that the widely used phenomenological non-locality parameter may underestimate the true microscopic non-locality for most nuclei. The discrepancy is most pronounced for ^{208}Pb , where β_v exceeds 1.0 fm at all energies in the full calculation. This finding has direct implications for transfer reaction calculations that employ the Perey correction factor, as an underestimated β would lead to inaccurate corrections.

The apparent tension between the larger microscopic β values found in this work and the long-standing empirical success of $\beta_{\text{PB}} = 0.85$ fm in describing nucleon-nucleus scattering deserves a clarifying remark. Three observations help reconcile this apparent contradiction. First, the Perey-Buck potential was originally constrained by neutron scattering on a single nucleus (^{208}Pb) at only two energies (7 and 14.5 MeV), and the value $\beta = 0.85$ fm is best understood as an effective parameter whose strength and geometry have been jointly readjusted to reproduce a limited

subset of elastic-scattering data, rather than as a faithful representation of the true non-locality of the underlying many-body self-energy. Second, in phenomenological optical potentials, the depth, radius, and diffuseness of the imaginary part are fitted simultaneously with β ; non-trivial correlations among these parameters allow different values of β to compensate one another at the level of observables, especially when only the angular distribution of elastic cross sections is constrained. Third, recent investigations of the separability of microscopic optical potentials by Arellano et al. [30, 31] have shown that the effective non-locality range emerging from a Perey-Buck-type Gaussian fit depends strongly on the spatial weight assigned to the volume versus surface regions, with bell-shape structures emerging in momentum space; this is consistent with our observation that the surface and volume β differ substantially in light nuclei.

3.6 Physical interpretation and systematics

The results presented above reveal a coherent physical picture that can be summarised as follows. The non-locality of the microscopic optical potential arises primarily from the dynamical self-energy $\Delta\Sigma(\omega)$, which describes the coupling of the incident nucleon to collective excitations of the target nucleus. The spatial extent of this coupling, and hence the non-locality parameter β , is governed by the overlap between single-particle wave functions and the phonon transition densities, mediated by the particle-vibration coupling vertex.

The velocity-dependent terms (t_1 and t_2) modify this vertex in two ways: (i) they alter the phonon properties through their contribution to the RPA residual interaction; (ii) they introduce momentum-dependent factors into the coupling matrix elements. Since these terms simulate the finite-range character of the N-N interaction, their removal effectively changes the range of the

residual interaction in coordinate space, leading to the observed changes in β . The mass dependence of this effect reflects the varying importance of volume versus surface contributions to the absorption as one moves from light to heavy nuclei.

The relatively weak influence of the spin-orbit term on β for light and medium-mass nuclei can be understood from the fact that the spin-orbit coupling primarily redistributes strength among different spin-orbit partners without significantly changing the overall radial extent of the coupling. Its effects are therefore more pronounced in spin-dependent observables than in the spin-averaged non-locality parameter. The exceptional behaviour of ^{208}Pb highlights, however, that in heavy nuclei, the interplay between the spin-orbit coupling and the large number of contributing phonon states can produce qualitatively different effects.

For light nuclei such as ^{16}O , there is a significant difference (~ 0.3 fm) between β_v and β_s , where $\beta_v < \beta_s$, reflecting the distinct physical mechanisms governing absorption in the nuclear interior (Pauli blocking, restricted phase space) versus the surface (strong coupling to low-lying collective states). For heavier nuclei, the values of β_v and β_s converge, which is consistent with the development of a well-defined nuclear interior where both volume and surface absorption mechanisms contribute comparably. The exception is ^{208}Pb , where $\beta_v > \beta_s$ in the full calculation, indicating that the interior non-locality is dominated by the coupling to a dense spectrum of collective excitations characteristic of this heavy doubly-magic nucleus.

4 Conclusions

We have performed a systematic investigation of the effects of the velocity-dependent (t_1 and t_2) and spin-orbit (W_0) terms of the Skyrme effective interaction on the non-locality parameter β of the

microscopic optical potential for neutron elastic scattering off the doubly closed-shell nuclei ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at incident energies below 20 MeV. The calculations employ the fully self-consistent particle-vibration coupling framework on top of Hartree-Fock plus random-phase approximation calculations with the SLy5 interaction. Our main findings are:

1. The velocity-dependent terms have a significant and mass-dependent impact on the non-locality parameter. For the light nucleus ^{16}O , removing these terms increases βv by approximately 0.27 fm, while for the heavy nucleus ^{208}Pb , the trend reverses, with βv decreasing by about 0.50 fm. This demonstrates that the finite-range simulation provided by the t_1 and t_2 terms plays a crucial role in shaping the non-local structure of the optical potential.

2. The spin-orbit interaction has a comparatively minor effect on the non-locality parameter in light and medium-mass nuclei, with changes typically less than 0.1 fm.

3. For all targets and energies considered, the extracted non-locality parameter of the imaginary part of the optical potential lies in the range 0.70–1.43 fm, with the majority of values systematically larger than the phenomenological Perey-Buck value of 0.85 fm. This suggests that the commonly adopted value may underestimate the true microscopic non-locality for most nuclei.

These results provide valuable microscopic constraints for the parameterisation of non-local optical potentials, which are essential for accurate nuclear reaction calculations.

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References

1. Feshbach H. Unified theory of nuclear reactions. *Annals of Physics*. 1958;5(4):357-390.
2. Dickhoff WH, Charity RJ. Recent developments for the optical model of nuclei. *Progress in Particle and Nuclear Physics*. 2019;105:252-299.
3. Bonaccorso A. Direct reaction theories for exotic nuclei: An introduction via semi-classical methods. *Progress in Particle and Nuclear Physics*. 2018;101:1-54.
4. Perey F, Buck B. A non-local potential model for the scattering of neutrons by nuclei. *Nuclear Physics*. 1962;32:353-380.
5. Titus LJ, Nunes FM, Potel G. Explicit inclusion of nonlocality in (d,p) transfer reactions. *Physical Review C*. 2016;93(1):014604.
6. Lovell AE, Nunes FM. Systematic uncertainties in direct reaction theories. *Journal of Physics G: Nuclear and Particle Physics*. 2015;42(3):034014.
7. Deltuva A. Three-body direct nuclear reactions: Nonlocal optical potential. *Physical Review C*. 2009;79(2):021602(R).
8. Timofeyuk N, Johnson R. Nonlocality in the adiabatic model of A(d,p)B reactions. *Physical Review C*. 2013;87(6):064610.
9. Timofeyuk N, Johnson R. Nonlocality in deuteron stripping reactions. *Physical Review Letters*. 2013;110(11):112501.
10. Ross A, Titus LJ, Nunes FM, Mahzoon MH, Dickhoff WH, Charity RJ. Effects of nonlocal potentials on (p, d) transfer reactions. *Physical Review C*. 2015;92(4):044607.
11. Quaglioni S, Navrátil P. Ab initio many-body calculations of n- ^3H , n- ^4He , p- $^3,^4\text{He}$, and n- ^{10}Be scattering. *Physical Review Letters*. 2008;101(9):092501.
12. Rotureau J, Danielewicz P, Hagen G, Nunes F, Papenbrock T. Optical potential from first principles. *Physical Review C*. 2017;95(2):024315.
13. Holt JW, Kaiser N, Miller GA. Microscopic optical potential for exotic isotopes from chiral effective field theory. *Physical Review C*. 2016;93(6):064603.
14. Whitehead T, Lim Y, Holt JW. Proton elastic scattering on calcium isotopes from chiral nuclear optical potentials. *Physical Review C*. 2019;100(1):014601.

15. Whitehead T, Lim Y, Holt JW. Global Microscopic Description of Nucleon-Nucleus Scattering with Quantified Uncertainties. *Physical Review Letters*. 2021;127(18):182502.
16. Bauge E, Delaroche JP, Girod M. Semimicroscopic nucleon-nucleus spherical optical model for nuclei with $A \geq 40$ at energies up to 200 MeV. *Physical Review C*. 1998;58(2):1118-1145.
17. Mahzoon MH, Charity RJ, Dickhoff WH, Dussan H, Waldecker SJ. Forging the link between nuclear reactions and nuclear structure. *Physical Review Letters*. 2014;112(16):162503.
18. Dickhoff WH, Van Neck D, Waldecker SJ, Charity RJ, Sobotka LG. Nonlocal extension of the dispersive optical model to describe data below the Fermi energy. *Physical Review C*. 2010;82(5):054306.
19. Mizuyama K, Ogata K. Self-consistent microscopic description of neutron scattering by ^{16}O based on the continuum particle-vibration coupling method. *Physical Review C*. 2012;86(4):041603(R).
20. Mizuyama K, Ogata K. Low-lying excited states of ^{24}O investigated by a self-consistent microscopic description of proton inelastic scattering. *Physical Review C*. 2014;89(3):034620.
21. Blanchon G, Dupuis M, Arellano HF, Vinh Mau N. Microscopic positive-energy potential based on the Gogny interaction. *Physical Review C*. 2015;91(1):014612.
22. Blanchon G, Dupuis M, Arellano HF. Prospective study on microscopic potential with Gogny interaction. *The European Physical Journal A*. 2015;51:165.
23. Blanchon G, Dupuis M, Bernard RN, Arellano HF. Asymmetry dependence of Gogny-based optical potential. *The European Physical Journal A*. 2017;53:88.
24. Hao TVN, Loc BM, Phuc NH. Low-energy nucleon-nucleus scattering within the energy density functional approach. *Physical Review C*. 2015;92(1):014605.
25. Nhan Hao TV, Nhu Le N, Koh M-H, Quang Hung N, Ngoc Duy N, Pham VNT, et al. Microscopic optical potential obtained from energy-density-functional approach for neutron-nucleus elastic scattering. *International Journal of Modern Physics E*. 2018;27(06):1850052.
26. Hoang Tung N, Quang Tam D, Pham VNT, Lam Truong C, Hao TVN. Effects of velocity-dependent and spin-orbit terms of the Skyrme interaction on neutron elastic scattering observables. *Physical Review C*. 2020;102(3):034608.
27. Hebborn C, Nunes FM, Potel G, Dickhoff WH, Holt JW, Atkinson MC, et al. Optical potentials for the rare-isotope beam era. *Physics G: Nuclear and Particle Physics*. 2023;50(6):060501.
28. Mau NV, Bouyssy A. Optical potential for low-energy neutrons: Imaginary potential for neutron- ^{40}Ca elastic scattering. *Nuclear Physics A*. 1976;257(2):189-220.
29. Bernard V, Van Giai N. Microscopic optical potential for ^{208}Pb in the nuclear structure approach. *Nuclear Physics A*. 1979;327(2):397-418.
30. Arellano HF, Blanchon G. On the separability of microscopic optical model potentials and emerging bell-shape Perey-Buck nonlocality. *The European Physical Journal A*. 2022;58(7):119.
31. Arellano HF, Blanchon G. Universal separable structure of the optical potential. *Physical Review C*. 2024;109(6):064609.
32. Vautherin D, Brink DM. Hartree-Fock calculations with Skyrme's interaction. I. Spherical nuclei. *Physical Review C*. 1972;5(3):626-47.
33. Chabanat E, Bonche P, Haensel P, Meyer J, Schaeffer R. A Skyrme parametrization from subnuclear to neutron star densities. Part II. Nuclei far from stabilities. *Nuclear Physics A*. 1998;635(1):231-56.
34. Colò G, Sagawa H, Bortignon PF. Effect of particle-vibration coupling on single-particle states: A consistent study within the Skyrme framework. *Physical Review C*. 2010;82(6):064307.
35. Cao L-G, Colò G, Sagawa H, Bortignon PF. Properties of single-particle states in a fully self-consistent particle-vibration coupling approach. *Physical Review C*. 2014;89(4):044314.
36. Colò G, Cao L, Van Giai N, Capelli L. Self-consistent RPA calculations with Skyrme-type interactions: The *skyrme_rpa* program. *Computer Physics Communications*. 2013;184(1):142-161.
37. Raynal J. Computer program DWBA98. NEA 1209/05; 1998.
38. National Nuclear Data Center. Experimental data taken from the National Nuclear Data Center. Brookhaven National Laboratory Online Data Service; 2025.